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The two-dimensional Heisenberg ferromagnet with various types of interactions: temperature dependence of magnetic parameters

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Abstract. The temperature dependences of the anisotropy constant, spin-wave stiffness and spontaneous magnetizations for the two-dimensional (2D) Heisenberg model with various types of anisotropy and interactions have been calculated with the use of the low-temperature perturbation (spin-wave) theory. We argue that this theory is effective within a large portion (if not the main part) of the ferromagnetic phase. On the contrary, the Polyakov renormalization procedure is found to be inapplicable for the ferromagnetic phase at any anisotropy. We consider both the easy-axis and the easy-plane cases, taking into account one-site and exchange anisotropy as well as the higher-order magnetic anisotropy and the dipole–dipole interaction. For realistic models the latter is found not to change essentially the temperature dependence of the magnetic parameters. In particular, we did not find the reorientation (easy-axis to easy-plane) phase transition due to dipole–dipole interaction predicted by Pescia and Pokrovsky. However, in general, the temperature dependence of the anisotropy constant proves to be much more sensitive to the type of the anisotropy and interactions than that of the spin stiffness and the spontaneous magnetization. In the easy-plane case the difference between the spin-wave stiffnesses for the in-plane and the normal polarizations is found to be extremely strongly temperature-dependent. At temperatures comparable with T_C this difference may be of the order of magnitude of the stiffness itself, while at $T = 0$ it is of relativistic origin. For the case of a very small anisotropy we propose as well a renormalization procedure to perform a partial summation of the perturbation series.

1. Introduction

Ultrathin transition-metal films have become a very active field of research recently (see e.g. [1]) because of many unusual magnetic properties and perspectives of research and applications. In theoretical interpretation of their properties it is natural to refer to the 2D Heisenberg model, and thus a better understanding of the properties of this model is quite important.

In this paper we develop a low-temperature perturbation theory to find the temperature dependence of magnetic parameters and to analyse the role of different interactions. We argue that this approach is reliable within a wide temperature region comparable with the ferromagnetic region itself. It is found that in this region the

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perturbation theory is better than a more sophisticated Polyakov renormalization procedure [2] used recently [3] for the anisotropic Heisenberg model.

The possibility of using the perturbation theory for the system in question was recognized, in fact, quite long ago, since it had been understood [4, 5] that, while long-range order is absent at $T \neq 0$ for the isotropic Heisenberg model, the anisotropy (or the dipole–dipole interaction [6]) restores the long-range order at low T, i.e. suppresses the thermal fluctuations. Several authors [4–8] have calculated the first term in the T series for the spontaneous magnetization; to this end one uses the harmonic part of the Hamiltonian only. But to calculate the next terms in this expansion as well as to investigate the T dependence of other magnetic parameters, e.g. of the anisotropy constant or of the spin stiffness, one has to take into account the anharmonicities and, for realistic models, it has never been done correctly to the best of our knowledge. This is surprising because for the 3D case the calculation of the terms of higher order in T is well known (see e.g. [9, 10]), and the 2D case is even simpler, in some respects, than the 3D one.

Indeed, it is the quantum-statistics effects that govern the temperature dependence of the spontaneous magnetization in a 3D system, while for 2D systems the use of classical statistics leads to a logarithmic error only as to the first-order term in the T dependence of the spontaneous magnetization; this is why several authors (see e.g. [3, 11, 12]) have used just classical statistics for the problem. Let us mention also that the classical results can be easily corrected to incorporate the quantum-statistics effects by an appropriate choice of the cut-off wavevector. It is not evident, of course, that the cut-off wavevector k_m, which enters the results of the harmonic theory as $\ln k_m$, plays the same role in the anharmonic theory. We shall see that sometimes it is, in fact, not the case and that the neglect of the quantum-statistics effects was one of the reasons for the erroneous conclusion made by Pescia and Pokrovsky [3] about the possibility of a reorientation phase transition due to the dipole–dipole interaction. But even in such a situation the classical results are useful, because, corrected in the above-mentioned way, they enable one to make quite reliable estimations. Let us mention also that it is far from simple to develop a consistent quantum theory of the low-T anharmonic effects because of the well known specific feature of 2D systems, namely formation of spin-wave bound states at any wavevector.

One can expect that the perturbation theory, being a low-temperature theory by virtue of its construction, is valid, in fact, up to temperatures comparable with the Curie temperature T_C. Indeed, the anisotropy is highly effective in suppressing the thermal fluctuations: it restores the long-range order up to temperatures that are much higher than the anisotropy energy (E_a). The reason is that the main role of the anisotropy is to fix the mean direction of the spontaneous magnetization, while the restoring force for local fluctuations of the spin direction is provided by the exchange interaction. These fluctuations, the spin waves, remain small within all the ferromagnetic phase because as they become large a phase transition occurs due to another type of fluctuation, the spin flips.

The spin flips that play the main role in the phase transition are extremely rare at low T and, to a much greater extent than the spin waves, within the main part of the ferromagnetic region. Recall the T dependence of the spontaneous magnetization for the 2D Ising model (which is due to the spin flips only): at $T = 0.5T_C$ it is about 0.2% less than at $T = 0$ (10% at $T = 0.8T_C$). As to the anisotropic Heisenberg model, the spin-flip activation energy at $T = 0$ is the same as for the Ising model and is equal to the exchange energy E_{exch}. True, owing to spin waves, it decreases as

T increases, but as T_C is less than in the Ising case one can expect that also for the Heisenberg model the spin flips are quite rare unless in the immediate vicinity of the phase transition. For both models the spin-flip concentration at low T is proportional to $\exp(E_{exch}/T)$ and this is why the spin flips are completely neglected by the low-T perturbation theory: it provides expansion in powers of T while all the derivatives of the above exponent are zero at $T = 0$. Thus the perturbation theory takes into account the spin waves only [5, 9, 10]. Now it is easy to realize that it is erroneous to use the Polyakov renormalization procedure [2] to calculate the T dependence of magnetic parameters in the ferromagnetic phase as has been done by Pescia and Pokrovsky [3]. Indeed, within this procedure, which has been proposed to treat the isotropic Heisenberg systems, no distinction is made between the spin waves and the spin flips. When applied to an anisotropic system it provides algebraic expressions for the T dependence of magnetic parameters. It means, in effect, that both the spin-wave and the spin-flip contributions are power-like within this procedure. This may well be correct for the paramagnetic phase at $T \gg T_C$ but is incorrect for $T < T_C$, and the perturbation theory that simply neglects the spin flips proves to be more exact for $T < T_C$ than the Polyakov procedure, which, in the case of an anisotropic system, overestimates the spin flips, ascribing to them power-in-T contributions. The above arguments about effectiveness of the perturbation theory are supported by our Monte Carlo calculations [13–15].

It is worth while to underline that even if they were formally correct the results of the Polyakov renormalization procedure would still be less useful for interpretation of the experimental data than the results of the perturbation theory. Indeed, this procedure provides a partial summation of the perturbation series; namely the most divergent terms, the 'leading' terms, are summarized. More specifically, the terms $[T\ln(E_{exch}/E_a)]^n$ are taken into account and others that contain the logarithm in a lower power are neglected. But in reality the characteristic value of E_{exch}/E_a is 10^2; anyway it is no more than 10^3. It means that the logarithm is never big enough to neglect the 'less important' terms, and there is no reason to believe that summation of the infinite series of 'leading' terms gives a more exact result than the calculation of several lowest-order perturbation theory terms, both leading and non-leading ones. It does not mean, of course, that this summation is not an interesting (albeit, at least at present, mainly methodical) problem.

Let us now mention papers by other authors devoted to the calculation of the T dependence of the magnetic parameters of 2D anisotropic Heisenberg systems. Khokhlachev [11] treated an easy-axis system without the in-plane anisotropy, i.e. considered the case when no spontaneous magnetization is present. Feigelman and Pokrovsky [12] have taken into account the dipole–dipole interaction for this model to calculate spontaneous magnetization and other magnetic parameters. We will abstain from criticism of this paper; let us mention only that for a real system the in-plane highest-order anisotropy is of much greater importance than the dipole–dipole interaction (see section 7). The easy-axis case has been investigated in the above cited paper by Pescia and Pokrovsky [3]. However, this paper is erroneous (see [16] and section 6).

Thus we were unable to find in the literature a consistent treatment of the T dependence of the magnetic parameters (outside the harmonic approximation) of the anisotropic Heisenberg system with realistic interactions. That is why we believe it is worth while to expose the relevant theory here.

The paper is organized as follows. In section 2 we describe the perturbation

theory techniques for, as an example, the case of one-site easy-axis anisotropy. In section 3 we calculate, for this case, the T dependence of the magnetic parameters and compare our results with those obtained by Pescia and Pokrovsky [3] with the help of the Polyakov renormalization procedure [2]. In section 4 we propose another renormalization procedure similar to that of Brezin and Zinn-Justin [17] and Nelson and Pelcovits [18] to calculate the T dependence of the magnetic parameters in the case of very small anisotropy. In sections 5 and 6 we consider more realistic Hamiltonians taking into account the second-order exchange anisotropy and the fourth-order one (section 5) and the dipole–dipole interaction (section 6). We compare the results of section 6 with those of [3]. In section 7 we consider easy-plane systems with fourth-order in-plane anisotropy and dipole–dipole interaction, and show that the situation here is quite similar to that of the easy-axis case with the exception of the gigantic temperature dependence of the spin-stiffness anisotropy. In section 8 we summarize the results of the paper and discuss their applicability to interpret experimental data as well as their relevance to other systems.

2. Method

Let us consider the classical Heisenberg Hamiltonian with the single-site anisotropy:

$$H = -\tfrac{1}{2}\sum_{i,j} J(i,j)\boldsymbol{m}(i)\boldsymbol{m}(j) - \Lambda\sum_{i} m_z^2(i) \tag{1}$$

where i, j numerate the lattice site and $\boldsymbol{m}$ is the unit-length vector. When calculating the partition function one has to take into account the fact that of the variables m_x, m_y, m_z there are only two independent ones because of the condition $m^2 = 1$. Let us consider non-interacting isolated classical spins. It is evident that the calculation of the partition function implies integration over the unit sphere for every lattice site. It is convenient instead to integrate over the area of the unit circle in the m_x, m_y plane. We shall consider m_x, m_y as components of the 2D vector $\boldsymbol{n}$. One has, of course, to integrate twice because there are two hemispheres. It is necessary to take into account the change of the surface element after projecting on the plane: $\sin\vartheta\,\mathrm{d}\vartheta\,\mathrm{d}\varphi \to (1-n^2)^{-1/2}\,\mathrm{d}n_x\,\mathrm{d}n_y$ and to replace m_z in the Hamiltonian by $+(1-n^2)^{1/2}$ for the upper hemisphere by $-(1-n^2)^{1/2}$ for the lower one. As a result the partition function is given by the expression:

$$Z = \sum_{\{s_i\}}\int \exp\left[-\tfrac{1}{T}\left(-\tfrac{1}{2}\sum_{i,j} J(i,j)\{\boldsymbol{n}(i)\boldsymbol{n}(j) + s_i s_j[1-n^2(i)]^{1/2}[1-n^2(j)]^{1/2}\}\right.\right.$$
$$\left.\left. + \Lambda\sum_i n^2(i)\right)\right]\prod_i \frac{\mathrm{d}\boldsymbol{n}(i)}{[1-n^2(i)]^{1/2}} \tag{2}$$

where s_i is the Ising variable, i.e. a number that is either $+1$ or -1, $\{s_i\}$ designates the summation over the values of every Ising variable, the integration over $\boldsymbol{n}$ runs over the unit circle and each s_i ranges independently over the values ± 1. As

$$\prod_i \frac{\mathrm{d}\boldsymbol{n}(i)}{[1-n^2(i)]^{1/2}} = \exp\left(-\frac{1}{2}\sum_i \ln[1-n^2(i)]\right)\prod_i \mathrm{d}\boldsymbol{n}(i) \tag{3}$$

one can formally incorporate the term $(T/2)\sum_i \ln[1-n^2(i)]$ into the Hamiltonian [17, 18]. It is the resulting 'effective' Hamiltonian that we shall consider below.

Within the spin-wave theory, as has been mentioned before, one puts $s_i = 1$ for every site and, within the continuous media approximation to which we shall restrict ourselves, the 'effective' Hamiltonian acquires the form:

$$H_{cm} = \int \frac{\Gamma}{2}\left((\nabla n_x)^2 + (\nabla n_y)^2 + [\nabla(1-n^2)^{1/2}]^2 + \lambda n^2 + \frac{T}{2a^2}\ln(1-n^2)\right) d\boldsymbol{r} \tag{4}$$

where $\lambda = \Lambda/a^2$, a is the lattice constant (a^{-2} is the number of the lattice sites per unit area) and $\Gamma = 0.25Jr$ in the case of nearest-neighbour interaction $J(i-j) = J$, r is the nearest-neighbour number. We see that the anharmonicity in the Hamiltonian is due to the replacement of m_z by $(1-n^2)^{1/2}$ and to the occurrence of $\ln(1-n^2)$.

Recall that the integration over $\boldsymbol{n}$ is over a restricted area (unit circle) and it may seem to be quite unfortunate because the perturbation theory implies the Gaussian integration as a zero approximation. However, as the fluctuations of $\boldsymbol{n}$ are small, one can extend the limit of the integration to infinity without making a big error: the corrections are proportional to $\exp(-\Delta/T)$ where Δ is of order of magnitude of T_C, i.e. they are of the same nature as those due to the spin flips. The small magnitude of the local fluctuations ($\langle n^2\rangle < 1$) is, of course, the main condition of applicability of the perturbation theory.

The harmonic (and T-independent) part of H_{cm} is

$$H_h = \int \left(\frac{\Gamma}{2}[(\nabla n_x)^2 + (\nabla n_y)^2] + \lambda n^2\right) d\boldsymbol{r} = \frac{1}{2}\sum_{\boldsymbol{k}}(2\lambda + \Gamma k^2)\boldsymbol{n}(\boldsymbol{k})\boldsymbol{n}(-\boldsymbol{k}) \tag{5}$$

where $\boldsymbol{n}(\boldsymbol{k})$ is the Fourier component of $\boldsymbol{n}(\boldsymbol{r})$ and the area of the plane is assumed to be equal to unity. From equation (5) one has

$$\langle \boldsymbol{n}(\boldsymbol{k})\boldsymbol{n}(-\boldsymbol{k})\rangle = 2T/(2\lambda + \Gamma k^2). \tag{6}$$

Then

$$\langle m_z\rangle = \langle(1-n^2)^{1/2}\rangle \simeq 1 - \tfrac{1}{2}\langle n^2\rangle = 1 - \frac{1}{2}\sum_{\boldsymbol{k}}\langle \boldsymbol{n}(\boldsymbol{k})\boldsymbol{n}(-\boldsymbol{k})\rangle$$

$$= 1 - \frac{T}{4\pi\Gamma}\ln\frac{\Gamma k_m^2}{2\lambda} \equiv 1 - \frac{T}{T_C^{sw}}. \tag{7}$$

The quantum effects can be taken into account approximately if in equation (7) one uses instead of the classical cut-off $k_m = \pi/a$ the quantum one defined by the condition $h\omega(k) = T$ where $\omega(k)$ is the spin-wave frequency. As $h\omega(k) = \Gamma S^{-1}a^2k^2$ (S is the spin number), one has $k_m^{qu} = a^{-1}(ST/\Gamma)^{1/2}$. Using the quantum cut-off one obtains logarithmic corrections only and that is why the classical approach is quite justified for small anisotropies. One sees from equation (7) that the characteristic 'spin-wave' temperature is T_C^{sw} and the condition of smallness of the local fluctuation is $T < T_C^{sw}$. It is reasonable to expect [11] that for small anisotropies T_C is of order of magnitude of T_C^{sw}.

The coefficients λ and Γ in equation (4) we shall call, correspondingly, the anisotropy coefficient and the spin-wave stiffness. The 'renormalized' coefficients are defined as follows. Let us fix a Fourier component $n(k_0)$ and $n(-k_0)$ (of course, $n(-k_0) = n^*(k_0)$) and calculate

$$F(n(k_0), n(-k_0)) \equiv -T \ln \int \exp\left(-\frac{1}{T} H(n(k_0), n(-k_0) \ldots)\right) \prod_k \mathrm{d}n(k) \tag{8}$$

where we do not integrate over $n(k_0)$, $n(-k_0)$. The expansion of $F(n(k_0), n(-k_0))$ in terms of its arguments contains, of course, a term proportional to $n(k_0)n(-k_0)$. Expanding the coefficient of this term in terms of k_0 one obtains

$$2\tilde{\lambda} + \tilde{\Gamma} k_0^2 + \ldots \tag{9}$$

the coefficient $\tilde{\lambda}$ and $\tilde{\Gamma}$ being what we call the renormalized anisotropy and spin-wave stiffness coefficients. To the harmonic approximation (see equation (5)) one obtains, evidently,

$$\tilde{\lambda} = \lambda \qquad \tilde{\Gamma} = \Gamma. \tag{10}$$

It is well known that equation (6) obtained in the Gaussian approximation proves to be exact if one replaces $2\lambda + \Gamma k_0^2$ by the coefficient at $n(k_0)n(-k_0)$ mentioned above, which has the meaning of the generalized stiffness. Therefore, as to calculation of $\langle |n(k)n(-k)| \rangle$ the anharmonic effects are reduced to renormalization of λ, Γ and, in principle, to appearance of terms of higher order in k. But as to the higher-order fluctuations one has, of course, to take into account the non-Gaussian contribution.

Using the conventional thermodynamic perturbation theory (see e.g. [19]) one obtains

$$F(n(k_0), n(-k_0)) = F_h(n(k_0)n(-k_0)) + \langle H_a(n(k_0), n(-k_0)) \rangle - \frac{1}{2T}[\langle H_a^2(n(k_0), n(-k_0)) \rangle - \langle H_a(n(k_0), n(-k_0)) \rangle^2] + \cdots \tag{11}$$

where $H_a = H_{cm} - H_h$ and the brackets $\langle\ \rangle$ designate the Gaussian averaging with the help of H_h. Let us mention that in our case H_a is a non-algebraic function of its variables, i.e. when represented as a series it contains an infinite number of terms.

3. Temperature dependence of magnetic parameters in the case of one-site easy-axis anisotropy

The lowest-order part of H_a (see equation (4)) reads:

$$H_{a,1} = \frac{\Gamma}{2} \int [n_x^2 (\nabla n_x)^2 + n_y^2 (\nabla n_y)^2 + 2 n_x n_y \nabla n_x \nabla n_y] \, \mathrm{d}r - \frac{T}{2a^2} \int n^2 \, \mathrm{d}r. \tag{12}$$

According to equation (11) to calculate $\tilde{\Gamma}$, to a first approximation, it is enough to consider the first term in the first integral in equation (12). In the Fourier representation it takes the form:

$$-\frac{\Gamma}{2} \sum_{k_1+k_2+k_3+k_4=0} k_3 k_4 n_x(k_1) n_x(k_2) n_x(k_3) n_x(k_4). \tag{13}$$

There are only two ways to single out the term containing $k_0^2 n_x(k_0) n_x(-k_0)$: put $k_3 = k_0$, $k_4 = -k_0$ or vice versa. Retaining these terms only we get

$$\frac{\Gamma}{2} k_0^2 n_x(k_0) n_x(-k_0) \sum_k n_x(k) n_x(-k). \tag{14}$$

Thus to a first approximation

$$\tilde{\Gamma} = \Gamma(1 + T/T_C^{sw}). \tag{15}$$

It is also elementary to find the first-order renormalization of the anisotropy coefficient. Now the second integral in equation (12) is also relevant. Putting $k_0 = 0$ we obtain instead of equation (14):

$$\frac{\Gamma}{2} n_x^2(0) \sum_k k^2 n_x(k) n_x(-k) - \frac{T}{2a^2} n_x^2(0). \tag{16}$$

After averaging one finds

$$\tilde{\lambda} = \lambda + \frac{1}{2}(\Gamma \sum_k k^2 \langle n_x(k) n_x(-k) \rangle - T \sum_k 1) = \lambda(1 - T/T_C^{sw}). \tag{17}$$

Using the above results for $\tilde{\Gamma}$ and $\tilde{\lambda}$ one can obtain the next-order term of the series for magnetization substituting $\tilde{\Gamma}$ and $\tilde{\lambda}$ for Γ and λ in equation (7) and taking into account the next term in the expansion of the square root in this equation. As a result one obtains

$$\langle m_z \rangle = 1 - T/T_C^{sw} - T^2/T^* T_C^{sw} \tag{18}$$

where $T^* = 4\pi\Gamma$. We see that, somewhat unexpectedly, the T^2 term is not of the order of magnitude of $(T/T_C^{sw})^2$ but is smaller as long as $T^* > T_C^{sw}$.

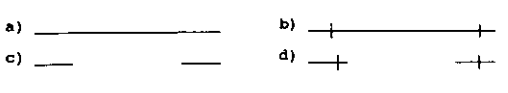

Figure 1. Lines in the graphs of perturbation theory expansion of $\tilde{\Gamma}$ and $\tilde{\lambda}$: (*a*) internal line, $\langle |n_x(k)|^2 \rangle$ or $\langle |n_y(k)|^2 \rangle$; (*b*) internal line, $k^2 \langle |n_x(k)|^2 \rangle$ or $k^2 \langle |n_y(k)|^2 \rangle$; (*c*) external lines, contribution to $\tilde{\lambda}$; (*d*) external lines, contribution to $\tilde{\Gamma}$.

Using equations (15) and (18) one can determine the temperature changes in the spin-wave dispersion law. Taking into account that the spin-wave frequency $\omega \sim \Gamma \langle m_z \rangle k^2 \equiv bk^2$ one finds that $b \sim 1 - (T/T_C^{sw})^2$. Thus a linear term is absent in the temperature dependence of b. It is remarkable that a similar situation takes place for 3D ferromagnets as well [20].

We display the T^2 contributions to $\tilde{\Gamma}$ and $\tilde{\lambda}$ with the help of diagrams (see figures 1–4), where we accept notations similar to those of [18]. After the relevant calculations one gets:

$$\tilde{\Gamma} = \Gamma \left[1 + \frac{T}{T_C^{sw}} - 2\frac{T^2}{T^* T_C^{sw}} - \zeta \left(\frac{T}{T^*} \right)^2 \right] \tag{19}$$

$$\tilde{\lambda} = \lambda \left[1 - \frac{T}{T_C^{sw}} + \left(\frac{T}{T_C^{sw}} \right)^2 - 2\frac{T^2}{T^* T_C^{sw}} + \psi \left(\frac{T}{T^*} \right)^2 \right] \tag{20}$$

where $\psi, \zeta \simeq 1$. Using these expressions one can find that

$$\langle m_z \rangle = 1 - T/T_C^{sw} - 2T^2/T^* T_C^{sw} - \tfrac{2}{3}(T/T_C^{sw})^3 \tag{21}$$

where the T^3 terms containing T/T^* are omitted as they are small in the small anisotropy case. It should be mentioned that to the T^2 approximation the spin stiffness becomes k-dependent at $T \neq 0$. It means, in fact, that the k^4 term in the spin-wave dispersion law, which we have neglected in the Hamiltonian (4), acquires a T dependence also. Nevertheless this term remains negligible at $k = k_m^{qu}$ if $T < (ST_C^{sw} T^* T_{an})^{1/3}$, where $T_{an} = \lambda a^2$. Although T_{an} is very small (several degrees), it enters the inequality in the power 1/3 only and for the typical magnitudes of Γ and λ the above condition is not very different from $T < T_C^{sw}$, which is the general condition of applicability of the perturbation theory. Let us mention also that as $T_C < T_C^{sw}$ (for small anisotropy $T_C \simeq T_C^{sw}/2$ [14, 15, 21]) the non-linear terms in equation (21) remain small even at $T \simeq T_C$.

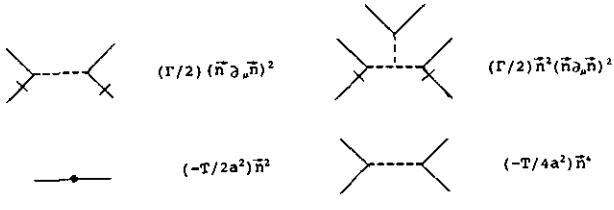

Figure 2. Vertices in the graphs of the perturbation theory expansion of $\tilde{\Gamma}$ and $\tilde{\lambda}$. Dashed lines separate n_x^2, n_y^2 corresponding to the same space coordinate.

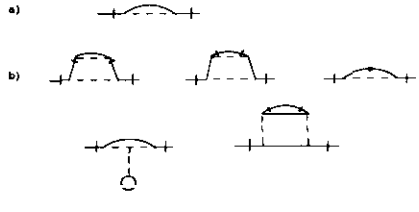

Figure 3. Perturbation theory graphs for $\tilde{\Gamma}$: (*a*) first order; (*b*) second order.

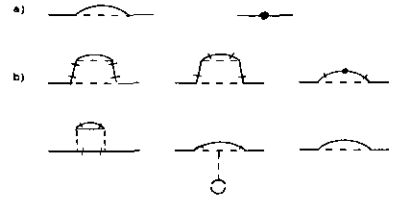

Figure 4. Perturbation theory graphs for $\tilde{\lambda}$: (*a*) first order; (*b*) second order.

To compare our results with those obtained by Pescia and Pokrovsky [3] with the help of the Polyakov renormalization procedure [2], one has to take into account the difference in the definition of the renormalized coefficients here and in [3]. Designating Γ_{pp} and λ_{pp} the coefficients of [3] one has:

$$\Gamma_{pp} = \langle m_z \rangle^2 \tilde{\Gamma} \qquad \lambda_{pp} = \langle m_z \rangle^2 \tilde{\lambda}. \tag{22}$$

Thus it follows from the results of [3] that

$$\tilde{\Gamma} = \Gamma[1 + T/T_C^{sw} + (T/T_C^{sw})^2] \tag{23}$$

$$\tilde{\lambda} = \lambda(1 - T/T_C^{sw}). \tag{24}$$

We see that in spite of the fact that the first-order terms coincide with those calculated within the perturbation theory, the second-order terms are quite different: the series for $\tilde{\Gamma}$ contains the second-order terms in T/T_C^{sw} while that for $\tilde{\lambda}$ does not. This observation means that one should discard the use of the Polyakov renormalization procedure for anisotropic systems at low T, no matter how small the anisotropy. The physical reasons for this situation are discussed at some length in section 1.

4. Temperature dependence of the magnetic parameters: renormalization results

Now we shall try to find the T dependence of $\tilde{\Gamma}$ and $\tilde{\lambda}$ using a renormalization procedure in a manner similar to that of Nelson and Pelcovits [18]. Our aim is to find the sums of the most important terms of the perturbation series for $\tilde{\Gamma}$ and $\tilde{\lambda}$. Let us recall once more that for our system these sums deviate from the exact values of $\tilde{\Gamma}$ and $\tilde{\lambda}$ because the exponential terms due to spin flips and the finiteness of limits of the integration over n are neglected. Taking into account also the importance of the 'non-leading' terms one sees that the finding of the sums being interresting *per se* is not expected to be important for interpretation of experimental data on the system in question.

The starting point of our calculations is the expression for the 'effective Hamiltonian' (in a different sense than in section 2). This Hamiltonian (or 'incomplete free energy') can be obtained if in equation (12) one leaves unintegrated not only $n(k_0)$ but all the Fourier components $n(k)$ with $k < q$. Because the integration over the Fourier components with $k > q$ means in fact averaging over the small-scale fluctuations, one may begin with the effective Hamiltonian that has the simplest form compatible with the symmetry (just as the Hamiltonian (4)) and describes the system with a bigger unit cell and an effective spin at an 'effective lattice site'. In our case one has to take into account that neglect of the spin flips, i.e. the assumption that the sign of the Ising variables s_i is the same for all the lattice sites, means that even at $\lambda = 0$ the uniaxial symmetry arises because now m_z is always positive. It means that unlike Hamiltonian (4) where the coefficients of $(\nabla m_x)^2$, $(\nabla m_y)^2$ and $(\nabla m_z)^2$ are the same, it is not the case for the effective Hamiltonian, and instead of the three first terms in equation (4) one now has

$$\tfrac{1}{2}\Gamma_q[(\nabla m_x)^2 + (\nabla m_y)^2] + \tfrac{1}{2}\Gamma_{1q}(\nabla m_z)^2 \tag{25}$$

where $m_{x,y,z}(r)$ contain the Fourier components with $k < q$ only. To get an expression of the form similar to that of equation (4) one has once more to take into account that m_x, m_y, m_z are not independent variables. But instead of the condition

$$m_x^2(i) + m_y^2(i) + m_z^2(i) = 1 \qquad \sum_{k \leqslant \pi a^{-1}} [|m_x(k)|^2 + |m_y(k)|^2 + |m_z(k)|^2] = 1 \tag{26}$$

one now has

$$\sum_{k<q} [|m_x(k)|^2 + |m_y(k)|^2 + |m_z(k)|^2] = a_q^2 < 1$$

$$\text{or} \qquad m_x^2(r) + m_y^2(r) + m_z^2(r) = a_q^2 \tag{27}$$

and the expression for the effective Hamiltonian reads:

$$H_{\text{eff}} = \int \left(\frac{\Gamma}{2}[(\nabla n_x)^2 + (\nabla n_y)^2] + \frac{\Gamma_{1q}}{2}[\nabla(a_q^2 - n^2)^{1/2}]^2 + \lambda_q n^2 \right) \mathrm{d}\boldsymbol{r} + \frac{Tq^2}{\pi^2} \int \ln(1 - n^2)\, \mathrm{d}\boldsymbol{r} \tag{28}$$

where $\boldsymbol{n}(\boldsymbol{r})$ contains the Fourier components with $k < q$ only. Equation (28) is, of course, approximate: it does not contain, in particular, the space derivatives of higher order in spite of the fact that, as we have seen already (section 3), the thermal fluctuations bring about the k dependence of the spin stiffness. However, this dependence reduces, in effect, to a renormalization of k_{m} while only $\ln k_{\text{m}}$ figures in the most important terms of the perturbation series, and thus this effect may be neglected.

The next step is to find the equations for Γ_q, Γ_{1q}, λ_q and a_q performing integration over $\boldsymbol{n}(\boldsymbol{k})$ with the k-vectors within the layer $q - \mathrm{d}q$ to q. Because of the smallness of the interval $\mathrm{d}q$, one can restrict oneself to the first-order perturbation theory. As a result one obtains:

$$\Gamma_{q-\mathrm{d}q} - \Gamma_q = -\mathrm{d}\Gamma_q = \Gamma_{1q} a_q^{-2} \langle n_x^2 \rangle_{\mathrm{d}q} = \Gamma_{1q} a_q^{-2} \frac{T}{2\lambda_q + \Gamma_q q^2} \frac{2\pi q\, \mathrm{d}q}{(2\pi)^2} \tag{29}$$

$$\lambda_{q-\mathrm{d}q} - \lambda_q = \tfrac{1}{2}\Gamma_{1q} a_q^{-2} q^2 \langle n_x^2 \rangle_{\mathrm{d}q} - T\frac{2\pi q\, \mathrm{d}q}{(2\pi)^2} = \frac{T}{2}\left(\frac{\Gamma_{1q} a_q^{-2} q^2}{2\lambda_q + \Gamma_q q^2} - 1 \right) \frac{2q\, \mathrm{d}q}{4\pi}. \tag{30}$$

One has, of course, to find also equations for Γ_{1q} and a_q. To this end one can look for renormalization of the coefficients in the terms of the fourth and sixth order in the effective Hamiltonian. It is fairly tedious but one can see that if one assumes that $\Gamma_{1q} a_q^{-2} = \Gamma_q$ one obtains a closed set of equations. We shall see that the solution of these equations is in agreement with the perturbation theory. Strictly speaking, it does not prove that this assumption is correct, but it shows that it is plausible, at least. One has

$$-\mathrm{d}\Gamma_q = (T/4\pi)\mathrm{d}\ln(2\lambda_q/\Gamma_q + q^2) \equiv (T/4\pi)\mathrm{d}\zeta \tag{31}$$

$$-\mathrm{d}\lambda_q = -(\lambda_q T/4\pi\Gamma_q)\mathrm{d}\zeta. \tag{32}$$

The solution of these equations is

$$\Gamma_q = C_1 - (T/4\pi)\zeta \tag{33}$$

$$\lambda_q = C_2/[C_1 - (T/4\pi)\zeta]. \tag{34}$$

The constants C_1 and C_2 are to be found from the condition that at $q = k_{\text{m}}$ there is no renormalization and $\Gamma_{q=k_{\text{m}}} = \Gamma$, $\lambda_{q=k_{\text{m}}} = \lambda$. As a result one finds for $q = 0$, i.e. for $\Gamma_{q=0} = \tilde{\Gamma}$, $\lambda_{q=0} = \tilde{\lambda}$:

$$\tilde{\Gamma} = \Gamma + (T/4\pi)\ln(\Gamma k_{\text{m}}^2/2\lambda) \tag{35}$$

$$\tilde{\lambda} = \lambda/[1 + (T/4\pi\Gamma)\ln(\Gamma k_{\text{m}}^2/2\lambda)]. \tag{36}$$

It is straightforward to check that these equations provide the same terms proportional to $T/T_{\text{C}}^{\text{sw}}$ and $(T/T_{\text{C}}^{\text{sw}})^2$ as the perturbation theory.

5. Exchange second-order and fourth-order anisotropies

There is no physical reason to reduce the second-order anisotropy to the one-site one. Let us take into account the exchange anisotropy as well. For the continuous-medium Hamiltonian one now has

$$H_{cm} = \frac{\Gamma}{2}\int[(\nabla m_x)^2 + (\nabla m_y)^2]\,d\mathbf{r} + \frac{\Gamma_1}{2}\int(\nabla m_z)^2\,d\mathbf{r} - \lambda\int m_z^2\,d\mathbf{r}. \tag{37}$$

Thus in equation (13) Γ has to be replaced by Γ_1. One finds instead of equations (15) and (17):

$$\tilde{\Gamma} = \Gamma\left(1 + \frac{T}{T_C^{sw}}\frac{\Gamma_1}{\Gamma}\right) \tag{38}$$

$$\tilde{\lambda} = \lambda\left(1 - \frac{T}{T_C^{sw}}\frac{\Gamma_1}{\Gamma}\right) + \frac{\Gamma_1 - \Gamma}{\Gamma}T\sum_k 1. \tag{39}$$

Because $\Gamma_1/\Gamma \simeq 1$ the change in the T dependence of $\tilde{\Gamma}$ is not essential. Of more interest is the change in the T dependence of $\tilde{\lambda}$. The second term in equation (39) containing a sum diverging at large k cannot be calculated within classical statistics and one has to use the quantum cut-off. As $k_m^{qu} \sim T^{1/2}$ this term is proportional to T^2. Thus, owing to quantum-statistics effects, equations (39) and (17) are practically equivalent to linear-in-T terms. We mention that for systems with small anisotropy the second term, even calculated within the classical statistics, is normally smaller than the main one: $\lambda(T/T_C^{sw})$. Indeed, if we assume realistically that $\Gamma_1 - \Gamma \simeq \lambda/a^2$ the second term in equation (39) is smaller than the main one as long as $T^* \gg T_C^{sw}$, i.e. for very small anisotropies it can be neglected. However, as mentioned above, for real systems the two sides of the inequality are of the same order of magnitude.

Let us take into account now the fourth order anisotropy, i.e. the term $K_1 n_x^2 n_y^2$. As long as this term does not contain the space derivatives it does not contribute, at least to the first order of the perturbation theory, to the T dependence of $\tilde{\Gamma}$ but does to that of $\tilde{\lambda}$. This contribution is

$$K_1 T/T_C^{sw} \tag{40}$$

and may be either positive or negative. As to the magnitude of K_1 the data are scarce for transition-metal films. It has been found for some of them that the fourth-order anisotropy constant is one order of magnitude less than the second-order one [21, 22].

Symmetry allows also the terms

$$K_{11}[(\nabla n_x)^2 n_y^2 + (\nabla n_y)^2 n_x^2] \qquad K_{22}[(\nabla n_x)^2 n_x^2 + (\nabla n_y)^2 n_y^2] \tag{41}$$

which provide a contribution to $\tilde{\Gamma}$:

$$\tfrac{1}{2}(K_{11} + K_{22})T/T_C^{sw}. \tag{42}$$

Estimating K_1 as λa^2, we see that as long as $\lambda a^2 \ll \Gamma$ this contribution can be neglected compared with that given by equation (20).

For the contribution to $\tilde{\lambda}$ one has

$$(K_{11} + K_{22}) \sum_{k} k^2 \frac{T}{2\lambda + \Gamma k^2} \simeq (K_{11} + K_{22}) \frac{T}{\Gamma} \sum_{k} 1. \tag{43}$$

This term is of the same structure as that in equation (39) and the above discussion is relevant here as well.

Thus taking account of the second-order exchange anisotropy and the fourth-order anisotropies does not lead to an appreciable change in the T dependence of the spin stiffness Γ and, thence, in the case of small anisotropy, of the T dependence of the magnetization. The anisotropy constant λ is more sensitive to the changes and can be essentially modified because of the contribution of the fourth-order anisotropy. The contribution of the exchange anisotropy is practically the same as that of the one-site one owing to the quantum-statistics suppression of short-wave fluctuations.

6. Easy-axis Heisenberg system with dipole–dipole interaction

The dipole–dipole term in the Hamiltonian has the conventional form:

$$\Omega \int \int \frac{\boldsymbol{m}(\boldsymbol{r})\boldsymbol{m}(\boldsymbol{r}') - 3[\boldsymbol{\nu}\boldsymbol{m}(\boldsymbol{r})][\boldsymbol{\nu}\boldsymbol{m}(\boldsymbol{r}')]}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \, \mathrm{d}\boldsymbol{r} \, \mathrm{d}\boldsymbol{r}' \tag{44}$$

where $\Omega = (g\mu S)^2$ and $\boldsymbol{\nu}$ is the unit vector pointing in the direction of $\boldsymbol{r} - \boldsymbol{r}'$. To use the independent variables n_x, n_y, one has to replace m_z by $(1 - n^2)^{1/2}$ as in section 2. For the harmonic part of the Hamiltonian (in the Fourier form) one has:

$$\Omega \sum_{\boldsymbol{k},\alpha,\beta} n_\alpha(\boldsymbol{k}) n_\beta(-\boldsymbol{k}) \Phi_{\alpha\beta}(\boldsymbol{k}) \tag{45}$$

where

$$\Phi_{\alpha\beta}(\boldsymbol{k}) = -3 \int \frac{\nu_\alpha \nu_\beta}{R^3} \, \mathrm{d}\boldsymbol{R} + \int \frac{(-3\nu_\alpha \nu_\beta + \delta_{\alpha\beta})[\exp(i\boldsymbol{k} \cdot \boldsymbol{R}) - 1]}{R^3} \, \mathrm{d}\boldsymbol{R}$$
$$= \Phi_{0\alpha\beta} + \Phi_{1\alpha\beta}(\boldsymbol{k}). \tag{46}$$

One sees from e.g. [3] that $\Phi_{0\alpha\beta} = -4\pi/a$ and from [6] that $\Phi_{1\alpha\beta}(k) = 2\pi k_\alpha k_\beta / k$ for $ka \ll 1$. As $\Phi_{1\alpha\beta}$ begins to be comparable with $\Phi_{0\alpha\beta}$ at $ka \simeq 1$ only and in this region of k-space the term Γk^2 in the coefficient of $n(k)n(-k)$ in equation (5) dominates because the exchange energy is much bigger than both the anisotropy and the dipole–dipole ones ($\Gamma \gg \lambda a^2$, $4\pi\Omega$), one can neglect $\Phi_{1\alpha\beta}(\boldsymbol{k})$ in all the integrals over k-space that enter the formulae for the magnetic parameters. Thus the essential contribution of the dipole–dipole interaction to the harmonic part of the Hamiltonian (as to calculation of the magnetic parameters) reduces to $-4\pi(\Omega/a) \int n^2(\boldsymbol{r}) \, \mathrm{d}\boldsymbol{r}$, which is, in fact, due to the demagnetizing field. We see that, to the harmonic approximation, the dipole–dipole interaction provides, in effect, a T-independent contribution to the anisotropy constant, the latter being now not λ but $\lambda_1 = \lambda - 4\pi\Omega/a$.

The contribution of the dipole–dipole interaction to the lowest-order anharmonic part of the Hamiltonian is

$$\frac{\Omega}{4}\int \frac{n^2(r)n^2(r') - \frac{1}{2}n^4(r) - \frac{1}{2}n^4(r')}{|r-r'|^3}\,dr\,dr'. \tag{47}$$

It is evident that to the first approximation this part does not contribute to $\tilde{\Gamma}$ but does to $\tilde{\lambda}$. To find this contribution it is convenient to represent $n(r)$ as $n_0+n'(r)$, where n_0 is the homogeneous part of $n(r)$, and retain the terms that are proportional to $n_{\alpha 0}^2$. It is enough to restrict ourselves to the term proportional to n_{x0}^2. One has

$$\Omega n_{x0}^2\int \frac{n_x'(r)n_x'(r') - \frac{1}{2}n_x'^2(r) - \frac{1}{2}n_x'^2(r')}{|r-r'|^3}\,dr\,dr' = \Omega n_{x0}^2\sum_k \psi(k)|n_x(k)|^2 \tag{48}$$

where [6]

$$\psi(k) = \int \frac{\exp(ik\cdot r)-1}{r^3}\,dr = -2\pi k. \tag{49}$$

Thus one obtains for the dipole–dipole contribution to $\tilde{\lambda}_1$:

$$-\pi\Omega T\sum_k \frac{k}{2\lambda_1+\Gamma k^2} = \frac{\Omega T}{2}\int \frac{k^2\,dk}{2\lambda_1+\Gamma k^2} \simeq -\frac{\Omega T}{2\Gamma}\int dk \simeq \frac{\Omega T^{3/2}}{2a\Gamma^{3/2}} \tag{50}$$

where the quantum cut-off vector has been used to estimate the integral in equation (50). Thus

$$\tilde{\lambda}_1 = \lambda_1(1-T/T_C^{sw}) - \Omega T^{3/2}/2a\Gamma^{3/2}. \tag{51}$$

We see that the anisotropy constant has, to a first approximation, the same temperature dependence as the anisotropy constant of the system without the dipole–dipole interaction. True, the latter provides a term proportional to $T^{3/2}$, but this term is of higher order in temperature and does not contain $\ln(\Gamma k_m^2)/2\lambda$. It means that, at least in the small anisotropy case, the second term in equation (51) can be neglected. Recall that it is just this case which has been treated in [3]. Thus the conclusion made there that the dipole–dipole interaction may lead to change of sign of the anisotropy constant at a temperature well below T_C proves to be unsubstantiated (for a detailed criticism of this paper, see [16]).

7. Easy-plane case

The easy-plane Heisenberg ferromagnet without dipole–dipole interaction and higher-order in-plane anisotropy is quite similar to the x–y model and thus there is no spontaneous magnetization and the phase transition is of the Berezinskiĭ–Kosterlitz–Thouless type. Evidently a higher-order in-plane anisotropy that fixes the direction of the magnetization in the plane stabilizes the long-range order, making the local fluctuations of the spin direction small at low T [4,5]. Less trivial is that the

dipole–dipole interaction brings about the same effect in the absence of the in-plane anisotropy [6]. Thus for a model of a 2D in-plane ferromagnet that takes into account the in-plane anisotropy and/or the dipole–dipole interaction, the application of the perturbation theory is quite possible and has been repeatedly performed to calculate the T dependence of the spontaneous magnetization [4–8]. In this section we calculate the T dependence of the spin stiffness and the anisotropy constant as well as discuss the relative role of the dipole–dipole interaction and of the in-plane anisotropy in the T dependence of the magnetic parameters.

Let us first neglect the dipole–dipole interaction and take into account the in-plane anisotropy. To be specific we consider the case of cubic anisotropy, but our results remain valid also for other symmetries. In the cubic case the in-plane anisotropy is of the fourth order and one has to add the term $Kn_x^2n_y^2$ to the Hamiltonian. For $K > 0$ the equilibrium direction of the spontaneous magnetization is along x or y axes. Let it be the x axis. As we now expect the fluctuations of m_y and m_z to be small, we choose these variables as independent ones, putting $m_x = (1 - m_y^2 - m_z^2)^{1/2}$. Thus to the harmonic approximation one has

$$\langle|m_y(\boldsymbol{k})|^2\rangle = T/(2K + \Gamma k^2) \qquad \langle|m_z(\boldsymbol{k})|^2\rangle = T/(2|\lambda| + \Gamma k^2). \tag{52}$$

Evidently λ is negative now because we consider the easy-plane case.

To the harmonic approximation the T dependence of the spontaneous magnetization is given by

$$\langle m_x\rangle = 1 - \frac{1}{2}\sum_{\boldsymbol{k}}[\langle|m_y(\boldsymbol{k})|^2\rangle + \langle|m_z(\boldsymbol{k})|^2\rangle] = 1 - T_C/T_{C,0}^{sw} \tag{53}$$

where

$$T_{C,0}^{sw} = 4\pi\Gamma/\ln[\Gamma k_m^2/2(K|\lambda|)^{1/2}]. \tag{54}$$

According to experimental data and numerical calculations (see e.g. [22, 23]) the fourth-order anisotropy constants for transition-metal films are smaller than the second-order ones by one order of magnitude. Therefore, as one sees from equation (54), the characteristic temperature for the change of spontaneous magnetization is practically the same for both the easy-axis and the easy-plane cases. Using the same method as in section 3 one has for the spin-wave stiffness:

$$\tilde{\Gamma}_{yy} = \Gamma\left(1 + \frac{T}{4\pi\Gamma}\ln\frac{\Gamma k_m^2}{2K}\right) \equiv \Gamma\left(1 + \frac{T}{T_{C,1}^{sw}}\right) \tag{55}$$

$$\tilde{\Gamma}_{zz} = \Gamma\left(1 + \frac{T}{4\pi\Gamma}\ln\frac{\Gamma k_m^2}{2K}\right) \equiv \Gamma\left(1 + \frac{T}{T_{C,2}^{sw}}\right). \tag{56}$$

We see that, owing to the anharmonicity, there arises a difference between $\tilde{\Gamma}_{yy}$ and $\tilde{\Gamma}_{zz}$ even in the case when one has $\Gamma_{yy} = \Gamma_{zz}$ in the Hamiltonian. The T dependence of the spin stiffness anisotropy $\tilde{\Gamma}_{yy} - \tilde{\Gamma}_{zz}$ proves to be unusually strong, really gigantic because already at relatively low T (say at $T/T_C^{sw} = 0.1$) this quantity becomes very big compared with its usual value at $T = 0$, which is normally $10^{-3}\Gamma$.

For the temperature dependence of $\tilde{\lambda}$ and $\tilde{K}$, the latter coefficient being defined as the term proportional to $n_y(k_0)n_y(-k_0)$ at $k_0 \to 0$ in the thermodynamic potential (9), one has:

$$\tilde{\lambda} = \lambda(1 - T/T_{C,0}^{sw}) \tag{57}$$

$$\tilde{K} = K(1 - T/T_{C,1}^{sw}) - KT/T_{C,0}^{sw}. \tag{58}$$

One can expect that the perturbation theory is applicable when $T/T_{C,1}^{sw} < 1$ because the fluctuations of m_y are the most important. However all the $T_{C,i}^{sw}$ are of the same order of magnitude for real systems.

To consider effects of dipole–dipole interaction one can use the same method as in section 6. One finds that the harmonic contribution to the Hamiltonian is

$$\Omega \sum_{k} [\psi_z(k)m_z(k)m_z(-k) + \psi_y(k)m_y(k)m_y(-k)] \tag{59}$$

where

$$\psi_z(k) = \frac{4\pi}{a} + \int \frac{\exp(ik \cdot R) - 1}{R^3} \, dR \simeq \frac{4\pi}{a} \tag{60}$$

and

$$\psi_y(k) = \frac{2\pi k_y^2}{k} \tag{61}$$

for $ka \ll 1$.

To the first-order approximation the formula for spontaneous magnetization reads

$$\begin{aligned} \langle m_x \rangle &= 1 - \frac{1}{2}\sum_{k} \langle |m_y(k)|^2 \rangle - \frac{1}{2}\sum_{k} \langle |m_z(k)|^2 \rangle \\ &\simeq 1 - \frac{T}{8\pi\Gamma} \ln \frac{\Gamma^2 k_m^2}{(4\pi\Omega)^2} - \frac{T}{8\pi\Gamma} \ln \frac{\Gamma k_m^2}{4\pi\Omega/a}. \end{aligned} \tag{62}$$

This equation is nothing more than the classical version of a formula due to Maleev [6]. Compared with equation (54) one can say that the dipole–dipole interaction provides effective anisotropy for both m_z and m_y fluctuations. For the first ones the effective anisotropy constant is equal to $2\pi\Omega/a$ and for the second it is $(4\pi\Omega)^2/2\Gamma$.

It seems quite reasonable in the case of transition-metal films to estimate $2\pi\Omega/a$ as equal within an order of magnitude to the anisotropy constant due to spin-lattice interactions [24]. Thus the effective anisotropy constant for the in-plane fluctuations may be estimated as $\lambda(\lambda a^2/\Gamma)$. We see that as $\lambda a^2 \ll \Gamma$ the anisotropy for in-plane fluctuations due to dipole–dipole interactions is very small and for physical systems under considerations the fourth-order in-plane anisotropy is much more important.

The lowest-order anharmonic part of the dipole–dipole interaction term in the Hamiltonian is

$$\frac{\Omega}{4} \int\int \frac{n^2(r)n^2(r') - \frac{1}{2}n^4(r) - \frac{1}{2}n^4(r')}{|r - r'|^3} (1 - 3\nu_x^2) \, dr \, dr' \tag{63}$$

where $\boldsymbol{n} = (m_y, m_z)$, and instead of equation (48) one has

$$\Omega n_{z0}^2 \sum_{\boldsymbol{k}} \psi(\boldsymbol{k}) |n_z(\boldsymbol{k})|^2 + \Omega n_{y0}^2 \sum_{\boldsymbol{k}} \psi(\boldsymbol{k}) |n_y(\boldsymbol{k})|^2 \tag{64}$$

where

$$\psi(\boldsymbol{k}) = \int \frac{\exp(i\boldsymbol{k} \cdot \boldsymbol{R}) - 1}{R^3} \, \mathrm{d}\boldsymbol{R} = 2\pi k. \tag{65}$$

One concludes once again that, owing to the effects of quantum statistics, the lowest-order contributions of the anharmonic part of the dipole–dipole interaction to the anisotropy constants is proportional to $T^{3/2}$. We see also that even in the absence of the in-plane anisotropy at $T = 0$ it appears at $T \neq 0$ due to dipole–dipole interaction. Physically it means, of course, that the latter contributes to the T dependence of the in-plane anisotropy constant.

8. Conclusion

We have used the low-T perturbation theory to analyse the T dependence of the magnetic parameters of the two-dimensional Heisenberg ferromagnet in the ferromagnetic phase. We have found that the competing Polyakov renormalization procedure is inapplicable for the problem, providing correctly only the linear-in-T terms.

For the sake of simplicity we have adduced the perturbation theory formulae for the case when $\ln(\Gamma k_m^2/2\lambda) \gg 1$. In this case one can neglect the influence of uncertainty of choice of k_m as well as corrections due to quantum statistics. However, for systems with realistic parameters this logarithm is never very big and one has to take this into account when comparing quantitatively the results of the analytical calculations with those of a real or computer experiment. In the first case one has to take into account the quantum-statistics effects, and it is straightforward to do so within the first orders of the perturbation theory. In the second case, to deal with results of classical Monte Carlo calculations, for example, one has to take into account the real spin-wave dispersion law instead of restricting oneself by the long-wave approximation. The results of the perturbation theory become model-dependent but no obstacles are seen to obtain them.

The formulae of this paper in spite of being of semiquantitative nature provide a general picture of the relative role of different interactions in the temperature dependence of the magnetic parameters. We see, in particular, that the characteristic temperature for the T dependence of the anisotropy constant is T_C^{sw} (or T_C) and no reorientation phase transition due to dipole–dipole interaction at $T \ll T_C$ is expected, unlike the statement of Pescia and Pokrovsky [3]. In general the role of the dipole–dipole interaction in the T dependence of spontaneous magnetization, spin stiffness and anisotropy constant is found to be insignificant for all the real systems. We have found also that the T dependence of the spin stiffness is determined mainly by the exchange interaction, and other interactions do not influence essentially this T dependence. But it is not the case for the anisotropy constant, which is much more sensitive to the nature of the non-exchange interactions. In the case of the easy-plane ferromagnet we have found that the difference between the spin-wave stiffness

for the in-plane and the normal magnetizations has unusually strong temperature dependence, and being very small at $T = 0$ (10^{-3} of the stiffness) becomes of its order of magnitude at T comparable with T_C.

In our theory we have neglected the spin flips, and this makes the theory inapplicable in the vicinity of a ferromagnetic phase transition where the spin flips play the leading role. But we argued in section 1 that the temperature region where spin flips do not influence essentially the T dependence of spontaneous magnetization is fairly big. These qualitative arguments are supported by Monte Carlo results by Garcia and Ribas [13] and by Serena *et al* [14].

The approach of this paper may be of some use also for the theory of structural phase transitions. Indeed, the situation when one has to consider two different types of thermal excitations, i.e. small vibrations (phonons or spin waves in the present paper) and the jumps over the barrier of a double-well potential (spin flips in the present paper) is quite typical in this field. One can speculate that in some cases the jumps over the barrier ('quasi spin flips') become important, as to T dependence of various parameters, in a close vicinity of the phase transitions only, while in a wider temperature range the T dependence is governed by the small lattice vibrations, which renormalize as well the activation energy of the 'quasi spin flips'. However, the small anisotropy case, which was of major interest within the present paper, is a specific feature of magnetic systems and we do not know a similar example for the structural ones.

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